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An analysis on fully developed laminar fluid flow and heat transfer in concentric annuli with moving cores

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Abstract—An analysis is made on the momentum and heat transfer between a fully developed *laminar* fluid flow and a moving core of fluid or solid body in an annular geometry. The annulus is concentric and both the wall surfaces are smooth. The heating condition is a uniform heat flux at either or both of the surfaces. The two fundamental solutions are obtained and the solutions for the inner and outer tubes for any heat flux ratio are then obtained through influence coefficients, which are evaluated from the fundamental solution from the definition. The effects of various parameters such as the relative velocity, the ratio, etc. on friction factor and Nusselt number are investigated.

1. INTRODUCTION

THIS IS part of a continuing study on the fluid flow and heat transfer in a concentric annulus with a moving core of solid body or fluid. In our previous study [1], we presented the solution on the problem of *turbulent* fluid flow and heat transfer in a concentric annulus with a moving core for a condition of a constant heat flux at the core only.

Problems involving fluid flow and heat transfer with a moving core of solid body or fluid in an annular geometry can be found in many manufacturing processes, such as extrusion, drawing and hot rolling, etc. In such processes, a hot plate or cylindrical rod continuously exchanges heat with the surrounding environment.

Another example which involves such fluid flow and heat transfer phenomena is a train travelling at high speed in a long tunnel (e.g. the 54 km long Seikan tunnel in Japan, the Channel Tunnel between England and France or the proposed Northumberland Strait undersea tunnel linking Prince Edward Island and New Brunswick in eastern Canada), or underground railways, where a significant amount of thermal energy may be transferred to the surroundings.

Our interest in the present problem is rooted in the inverted annular film boiling [2] which may occur during the emergency core cooling of nuclear fuel channels. For such cases, the fluid flow involved can be either *laminar* or *turbulent* flow and there seems to be few reliable predictions for momentum and heat transfer available in the literature.

The case of the laminar and turbulent flow and heat transfer in the *boundary layer* on a continuous moving surface was studied by Tsou *et al.* [3].

In this paper, the momentum and heat transfer between a fully developed laminar fluid flow and a moving core of fluid or solid body in a *concentric annular geometry* is studied analytically.

The mathematical development of the analysis is straightforward and the velocity and temperature profiles are obtained from basic momentum and energy differential equations. The necessary boundary conditions are acquired from the heat flux at each of the two wall surfaces.

Since the energy equation is linear and homogeneous, superposition methods are used to obtain solutions for asymmetric heating by adding other solutions [4]. The two fundamental solutions obtained in the study are:

Case (A): the inner wall only heated with the outer insulated;

Case (B): the outer wall only heated with the inner insulated.

The solution for the inner and outer tubes for any heat flux ratio is then obtained through influence coefficients, which are evaluated from the fundamental solution from the definition.

The resulting momentum and heat transfer are discussed in terms of various parameters, namely the relative velocity, the radius ratio and fluid Reynolds number.

NOMENCLATURE

a	thermal diffusivity
f	friction factor
h	heat transfer coefficient
k	thermal conductivity
Nu	Nusselt number
P	pressure
q	heat flux
r	radial coordinate
$\bar{r}$	r/R_o
R	radius
Re	Reynolds number, $u_m \cdot 2(R_o - R_i)/\nu$
T	temperature
T^*	$(T_{R_i} - T)/\lvert T_{R_o} - T_{R_i}\rvert$
u	velocity
U	core velocity
U^*	U/u_m
x	axial coordinate.

Greek symbols

α	radius ratio, R_i/R_o
ζ	$(r-R_i)/(R_o-R_i)$
θ^*	influence coefficients
μ	viscosity
ν	kinematic viscosity
ρ	density.

Subscripts

b	bulk
crit	critical
i	inner
ii	constant heat rate at the inner wall with the outer insulated
m	average
o	outer
oo	constant heat rate at the outer wall with the inner insulated
R_i	inner radius
R_o	outer radius.

Constants

B	$(\alpha^2 - 1)/\ln \alpha$
M	$1 + \alpha^2 - B$
E	$(\alpha^2 - \frac{1}{2}B)/(\alpha^2 - 1)$
w	$(1 - \frac{1}{2}U^*)/(1 - EU^*)$
B^*	wB.

2. ANALYSIS

The assumptions used in the analysis are (see Fig. 1):

(i) The annulus is concentric and both the wall surfaces are smooth. The heating condition is a uniform heat flux at either or both of the surfaces.

(ii) The flow is incompressive and *laminar*. Velocity and temperature fields in the annulus are fully developed.

(iii) The viscous dissipation term is neglected.

2.1. *Momentum transfer*

With the assumptions described above, the governing momentum equation is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = \frac{1}{\mu}\frac{dP}{dx}. \quad (1)$$

Applying the boundary conditions

$$u = U \quad \text{at} \quad r = R_i \quad (2)$$

$$u = 0 \quad \text{at} \quad r = R_o \quad (3)$$

we easily obtained the velocity profile as

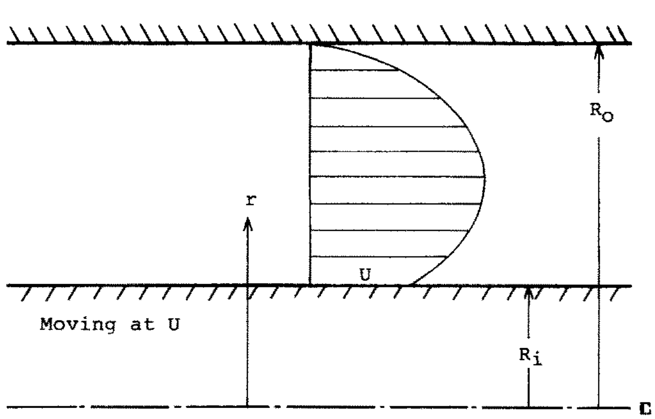

FIG. 1. Idealized model.

$$u = \frac{2u_m}{M}[1-EU^*][1-(\bar{r})^2+B^*\ln(\bar{r})] \quad (4)$$

where u_m is the average velocity, given as

$$u_m = \frac{R_o^2}{4\mu}\left(-\frac{dP}{dx}\right)\frac{M}{2}+EU. \quad (5)$$

Now that the velocity profile is known, we can obtain the friction factor, f, from the definition as

$$f = \frac{(R_o-R_i)}{\rho u_m}\left(-\frac{dP}{dx}\right) = \frac{16}{Re}\left[\frac{(1-\alpha)^2}{M}\right][1-EU^*]. \quad (6)$$

2.2. *Heat transfer*

(i) Cases for one wall only heated and the other insulated

The governing energy equation is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{u}{a}\left(\frac{dT_b}{dx}\right). \quad (7)$$

With the boundary conditions given as

	Case A: (ii)	Case B: (oo)
$r = R_i$	$T = T_{R_i}$	$\frac{\partial T}{\partial r} = 0$
$r = R_o$	$\frac{\partial T}{\partial r} = 0$	$T = T_{R_o}$

the temperature distribution is readily obtained as

$$T = A\left[\frac{(1-B^*)}{4}(\bar{r})^2-\frac{1}{16}(\bar{r})^4+\frac{B^*}{4}(\bar{r})^2\ln(\bar{r})\right] + C_1\ln(\bar{r})+C_2 \quad (8)$$

where

$$A \equiv \frac{R_o^2}{a}\left(\frac{dT_b}{dx}\right)\left(\frac{2u_m}{M}\right)(1-EU^*)$$

and the constants C_1 and C_2 are:

Case A: (ii)

$$C_1 = -\frac{A}{4}(1-B^*)$$

and

$$C_2 = T_{R_i}-A\left[\frac{(1-B^*)}{4}\alpha^2-\frac{\alpha^4}{16}+\frac{\alpha^2}{4}B^*\ln\alpha-\frac{(1-B^*)}{4}\ln\alpha\right]$$

Case B: (oo)

$$C_1 = -A\left[\frac{(2-B^*)}{4}\alpha^2-\frac{1}{4}\alpha^4+\frac{B^*}{2}\alpha^2\ln\alpha\right]$$

and

$$C_2 = T_{R_o}-A\frac{(3-4B^*)}{16}.$$

Now that the temperature distribution across the flow channel is known, Nusselt numbers, Nu_{ii} and Nu_{oo}, are calculated from the definition.

The Nusselt numbers are defined as

$$Nu_{jj} \equiv h_{jj}\cdot 2(R_o-R_i)/k \quad (9)$$

where jj = ii for Case A, and jj = oo for Case B.

The heat transfer coefficients, h_{ii} and h_{oo}, are defined, respectively, as

$$q_{R_i} \equiv h_{ii}(T_{R_i}-T_b) = -k\left.\frac{\partial T}{\partial r}\right|_{R_i} \quad (10)$$

and

$$q_{R_o} \equiv h_{oo}(T_{R_o}-T_b) = k\left.\frac{\partial T}{\partial r}\right|_{R_o}. \quad (11)$$

The bulk temperature, T_b, is defined as

$$T_b = \int_{R_i}^{R_o} ruT\,dr \Big/ \int_{R_i}^{R_o} ru\,dr \quad (12)$$

Nusselt numbers:

$$Nu_{jj} = \frac{2(1+\alpha)(1-\alpha)^2[1+(2w-1)\alpha^2-wB]^2}{FH_{jj}} \quad (13)$$

Case A: Nu_{ii} (jj = ii), $F = \alpha$ and

$$H_{ii} \equiv \left\{\frac{1}{B}-\left(\frac{25}{24}+2w\right)+\left(\frac{22}{9}+w\right)wB-\frac{11}{8}w^2B^2\right\} + \left\{\left(\frac{23}{24}-2w\right)-\left(\frac{14}{9}-2w\right)wB+\frac{5}{8}w^2B^2\right\}\alpha^2 - \left\{\left(\frac{13}{24}-2w+w^2\right)-\left(\frac{4}{9}-\frac{3}{2}w\right)wB\right\}\alpha^4 + \left(\frac{1}{8}-\frac{2}{3}w+w^2\right)\alpha^6 \quad (14)$$

Case B: Nu_{oo} (jj = oo), $F = 1$ and

$$H_{oo} = \left(\frac{11}{24}-\frac{19}{18}wB+\frac{5}{8}w^2B^2\right) - \left\{\left(\frac{61}{24}-3w\right)-\left(\frac{40}{9}-4w\right)wB+\frac{11}{8}w^2B^2\right\}\alpha^2 + \left\{\frac{[2(1-w)-(1-2w)\alpha^2]^2}{B}+\left(\frac{47}{24}-10w+7w^2\right) - \left(\frac{37}{18}-\frac{11}{2}w\right)wB\right\}\alpha^4 - \left(\frac{3}{8}-\frac{13}{3}w+7w^2\right)\alpha^6. \quad (15)$$

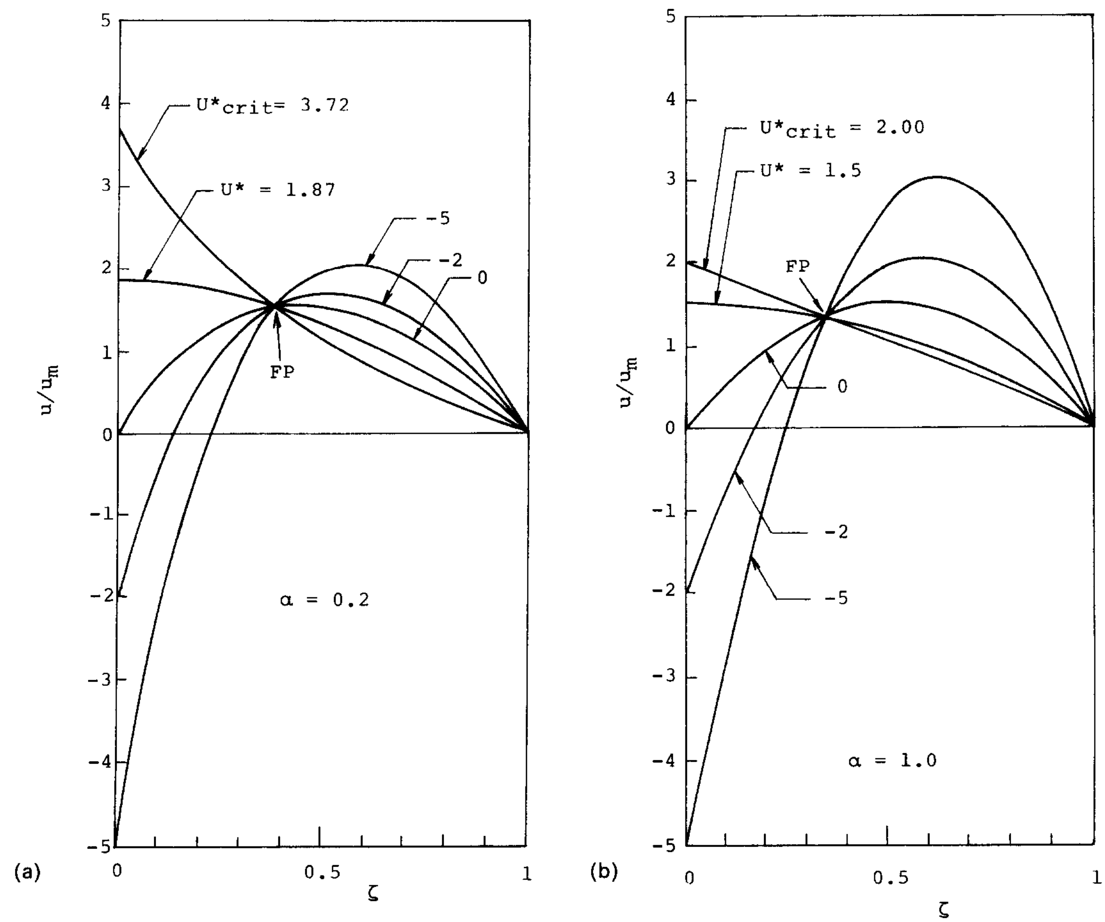

FIG. 2. Velocity profiles.

(ii) Cases for both walls heated independently

For the cases where both wall surfaces are heated independently, the Nusselt numbers on the two surfaces for any heat flux ratio may be calculated, utilizing the superposition method [4]. This is because the governing energy equation for the present study is linear and homogeneous. The Nusselt numbers for asymmetric heating are then obtained through influence coefficients [5] as

$$Nu_j = \frac{Nu_{jj}}{1-\theta_j^*(q_{R_k}/q_{R_j})} \tag{16†}$$

where j = i then k = o and j = o then k = i. The Nusselt numbers, Nu_i and Nu_o, are defined as

$$Nu_j \equiv \frac{h_j \cdot 2(R_o - R_i)}{k}. \tag{17}$$

The heat transfer coefficient is defined as

$$h_j \equiv q_{R_j}/(T_{R_j} - T_b) \tag{18}$$

where j = i or o.

† q_{R_i} and q_{R_o} are defined as positive into the fluid.

‡ The critical relative velocity, U^*_{crit}, is the value of U^* at $f = 0$, i.e. $(-dP/dx) = 0$. $U^*_{crit} = 1/E$ from equation (6).

The influence coefficients, θ_i^* and θ_o^*, are defined as [4]

$$\theta_j^* \equiv \frac{(T_b - T_{R_j})_{kk} \Big/ \left[\frac{q_{R_k}\cdot 2(R_o - R_i)}{k}\right]}{(T_{R_j} - T_b)_{jj} \Big/ \left[\frac{q_{R_j}\cdot 2(R_o - R_i)}{k}\right]} \tag{19}$$

where for Case A : j = i and k = o and for Case B : j = o and k = i.

The final derivation of θ_i^* and θ_o^* is given in the Appendix.

3. RESULTS AND DISCUSSION

The ranges of parameters considered are:

the radius ratio:	$0 \leqslant \alpha \leqslant 1$
the relative velocity:	$-5 \leqslant U^* \leqslant U^*_{crit}$.‡

Therefore, the discussion of the results in this paper is confined to the fluid flow with $f \geqslant 0$ and $Re > 0$. The results of the solutions are presented in Tables 1 and 2.

Table 1. Friction factors, Nusselt numbers and influence coefficients, fully developed laminar flow in concentric annuli with moving cores; $U^* \geqslant 0$

U^*	α	$f\,Re$	Nu_{ii}	Nu_{oo}	θ_i^*	θ_o^*
0.0	0.01	20.03	54.02	4.69	7.061	0.0061
	0.05	21.57	17.81	4.79	2.183	0.0294
	0.10	22.34	11.91	4.83	1.384	0.0562
	0.20	23.09	8.50	4.88	0.904	0.1039
	0.40	23.68	6.58	4.98	0.602	0.1822
	0.60	23.90	5.91	5.10	0.474	0.2455
	0.80	23.98	5.58	5.24	0.398	0.2991
	1.00	24.00	5.38	5.38	0.346	0.3462
0.25	0.01	19.49	54.21	4.65	7.202	0.0062
	0.05	20.68	17.98	4.71	2.266	0.0297
	0.10	21.19	12.09	4.73	1.456	0.0569
	0.20	21.54	8.72	4.74	0.968	0.1053
	0.40	21.58	6.87	4.80	0.657	0.1837
	0.60	21.41	6.24	4.90	0.523	0.2461
	0.80	21.20	5.95	5.02	0.442	0.2985
	1.00	21.00	5.78	5.16	0.386	0.3441
0.50	0.01	18.94	54.40	4.60	7.342	0.0062
	0.05	19.79	18.15	4.63	2.349	0.0300
	0.10	20.03	12.28	4.62	1.528	0.0575
	0.20	19.98	8.96	4.60	1.032	0.1061
	0.40	19.47	7.17	4.62	0.713	0.1838
	0.60	18.92	6.60	4.70	0.572	0.2445
	0.80	18.43	6.35	4.81	0.486	0.2948
	1.00	18.00	6.22	4.94	0.426	0.3382
1.0	0.01	17.86	54.77	4.50	7.619	0.0063
	0.05	18.02	18.49	4.48	2.514	0.0304
	0.10	17.72	12.67	4.42	1.673	0.0583
	0.20	16.88	9.45	4.33	1.162	0.1066
	0.40	15.27	7.83	4.28	0.826	0.1805
	0.60	13.95	7.40	4.32	0.673	0.2356
	0.80	12.88	7.27	4.40	0.577	0.2798
	1.00	12.00	7.24	4.52	0.509	0.3172
2.0	0.01	15.68	55.54	4.32	8.160	0.0063
	0.05	14.48	19.20	4.17	2.838	0.0308
	0.10	13.09	13.50	4.02	1.957	0.0583
	0.20	10.67	10.53	3.83	1.419	0.1031
	0.40	6.86	9.40	3.65	1.052	0.1634
	0.60	4.00	9.41	3.62	0.874	0.2018
	0.80	1.78	9.67	3.67	0.755	0.2291
	1.00	0.00	10.00	3.75	0.667	0.2500
3.0	0.01	13.51	56.31	4.14	8.684	0.0064
	0.05	10.93	19.94	3.88	3.151	0.0307
	0.10	8.46	14.38	3.66	2.234	0.0568
	0.20	4.46	11.75	3.37	1.667	0.0956
	0.40	−1.55	11.35	3.11	1.259	0.1379
	0.60	−5.95	12.09	3.04	1.038	0.1566
	0.80	−9.32	13.04	3.05	0.878	0.1644
	1.00	−12.00	14.00	3.11	0.750	0.1667
4.0	0.01	11.34	57.08	3.97	9.189	0.0064
	0.05	7.38	20.70	3.61	3.452	0.0301
	0.10	3.84	15.33	3.32	2.497	0.0541
	0.20	−1.75	13.14	2.97	1.894	0.0855
	0.40	−9.96	13.74	2.65	1.408	0.1088
	0.60	−15.90	15.52	2.56	1.088	0.1076
	0.80	−20.42	17.40	2.55	0.820	0.0963
	1.00	−24.00	19.09	2.59	0.591	0.0802
5.0	0.01	9.17	57.85	3.80	9.674	0.0064
	0.05	3.84	21.48	3.36	3.737	0.0292
	0.10	−0.79	16.33	3.02	2.740	0.0506
	0.20	−7.97	14.69	2.62	2.081	0.0742
	0.40	−18.37	16.55	2.28	1.433	0.0788
	0.60	−25.85	19.43	2.17	0.885	0.0592
	0.80	−31.53	21.83	2.15	0.385	0.0303
	1.00	−36.00	23.33	2.18	−0.028	−0.0026

Table 2. Friction factors, Nusselt numbers and influence coefficients, fully developed laminar flow in concentric annuli with moving cores; $U^* < 0$

U^*	α	$f\,Re$	Nu_{ii}	Nu_{oo}	θ_i^*	θ_o^*
−0.25	0.01	20.57	53.83	4.74	6.920	0.0061
	0.05	22.45	17.64	4.87	2.099	0.0290
	0.10	23.50	11.72	4.94	1.311	0.0553
	0.20	24.64	8.28	5.02	0.841	0.1020
	0.40	25.78	6.31	5.16	0.548	0.1793
	0.60	26.38	5.60	5.30	0.427	0.2424
	0.80	26.76	5.24	5.45	0.356	0.2964
	1.00	27.00	5.02	5.61	0.308	0.3439
−0.50	0.01	21.11	53.46	4.79	6.777	0.0060
	0.05	23.34	17.48	4.95	2.015	0.0286
	0.10	24.66	11.54	5.05	1.239	0.0542
	0.20	26.19	8.07	5.16	0.777	0.0995
	0.40	27.88	6.06	5.34	0.495	0.1746
	0.60	28.87	5.31	5.50	0.380	0.2365
	0.80	29.53	4.93	5.67	0.315	0.2897
	1.00	30.00	4.69	5.83	0.271	0.3368
−1.0	0.01	22.20	53.26	4.88	6.489	0.0059
	0.05	25.11	17.15	5.12	1.847	0.0275
	0.10	26.97	11.19	5.26	1.094	0.0514
	0.20	29.30	7.67	5.45	0.652	0.0927
	0.40	32.09	5.58	5.70	0.391	0.1598
	0.60	33.85	4.80	5.90	0.291	0.2152
	0.80	35.08	4.38	6.09	0.236	0.2631
	1.00	36.00	4.12	6.27	0.201	0.3060
−2.0	0.01	24.37	52.52	5.08	5.902	0.0057
	0.05	28.66	16.52	5.44	1.506	0.0248
	0.10	31.60	10.53	5.68	0.806	0.0435
	0.20	35.51	6.94	5.98	0.409	0.0706
	0.40	40.50	4.78	6.35	0.197	0.1048
	0.60	43.79	3.95	6.60	0.128	0.1285
	0.80	46.18	3.51	6.81	0.096	0.1484
	1.00	48.00	3.23	7.00	0.077	0.1667
−3.0	0.01	26.55	51.78	5.27	5.303	0.0054
	0.05	32.20	15.92	5.76	1.164	0.0211
	0.10	36.22	9.91	6.07	0.522	0.0320
	0.20	41.72	6.30	6.43	0.178	0.0363
	0.40	48.91	4.12	6.80	0.021	0.0137
	0.60	53.74	3.30	7.02	−0.015	−0.0195
	0.80	57.28	2.86	7.20	−0.025	−0.0513
	1.00	60.00	2.59	7.37	−0.028	−0.0789
−4.0	0.01	28.72	51.04	5.47	4.692	0.0050
	0.05	35.75	15.33	6.05	0.822	0.0162
	0.10	40.85	9.34	6.40	0.243	0.0167
	0.20	47.93	5.73	6.73	−0.042	−0.0099
	0.40	57.32	3.59	6.94	−0.139	−0.1074
	0.60	63.69	2.79	7.03	−0.141	−0.2132
	0.80	68.38	2.37	7.13	−0.129	−0.3104
	1.00	72.00	2.12	7.24	−0.116	−0.3966
−5.0	0.01	30.89	50.32	5.66	4.070	0.0046
	0.05	39.30	14.78	6.32	0.480	0.0103
	0.10	45.47	8.81	6.65	−0.029	−0.0022
	0.20	54.14	5.24	6.83	−0.250	−0.0651
	0.40	65.73	3.15	6.73	−0.283	−0.2419
	0.60	73.64	2.39	6.63	−0.251	−0.4188
	0.80	79.49	2.00	6.62	−0.219	−0.5789
	1.00	84.00	1.76	6.67	−0.191	−0.7222

The predicted velocity profiles in the annular ducts with the moving cores at various values of the relative velocity are shown in Figs. 2(a) and (b) for the values of $\alpha = 0.2$ and 1, respectively. It is interesting to note that once the value of α is given, all velocity profiles go through a fixed single point, FP, regardless of the value of U^*. Point FP moves toward the inner wall with increasing values of α, accompanied by a decrease in the value of u/u_m. In the region between FP and $\zeta = 1$ (the outer wall), it was also observed that u/u_m increases with decreasing U^* and that for $\alpha > 0.2$ and $U^* < 0$, there exists a maximum value of u/u_m.

The predicted friction factor in terms of $f \cdot Re$ for the case of $U^* = 0$ (both walls stationary) is shown

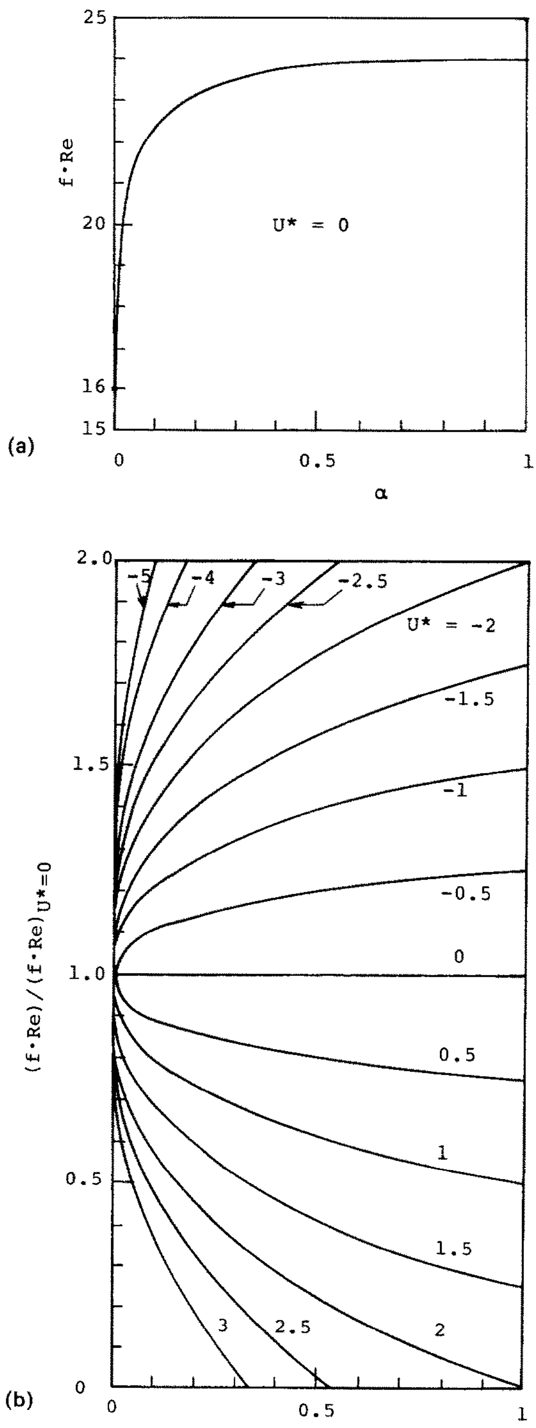

FIG. 3. Friction factors.

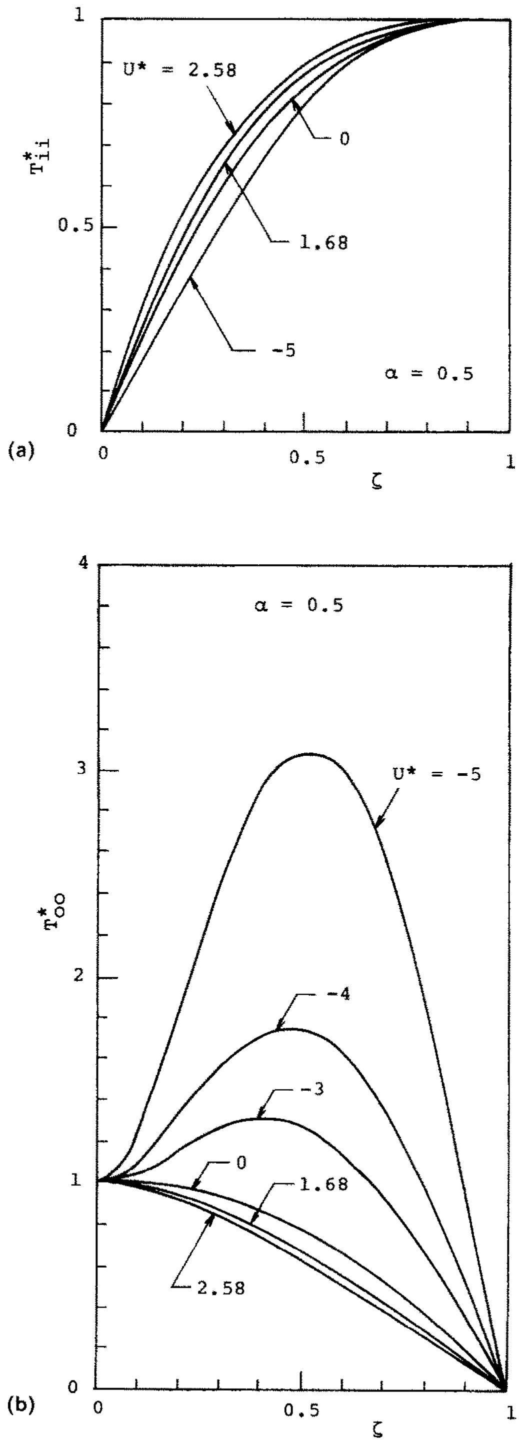

FIG. 4. Temperature profiles.

in Fig. 3(a), while Fig. 3(b) illustrates the effects of α and U^* on $(f \cdot Re)$, normalized by the value of $(f \cdot Re)$ for the case of $U^* = 0$. The effect of the relative velocity, U^*, is that for $U^* < 0$, the ratio $(f \cdot Re)/(f \cdot Re)_{U^*=0}$ is always greater than unity, whereas it is always less than unity for $U^* > 0$. Figure 3(b) clearly demonstrates that the effect of U^* is greatest at $\alpha = 1$ (parallel plates flow) and diminishes at $\alpha = 0$ (circular pipe flow). This is because with decreasing values of α, the role of the shear stress of the moving core surface on the overall pressure drop becomes less important.

Representative non-dimensional temperature profiles in a concentric annulus ($\alpha = 0.5$) with the moving core for Case A (the inner core wall only heated) and Case B (the outer wall only heated) are shown in Figs. 4(a) and (b), respectively. For Case A, the temperature gradient at the heated inner wall increases with an increase in U^*, implying that the heat transfer rate increases with increasing U^*. On the other hand, for Case B, the temperature gradient at the heated outer wall decreases with an increase in U^*, implying that the heat transfer deteriorates with increasing U^*. It is also seen that for $U^* < 0$, a maximum value (in dimensional T, a minimum value) exists for Case B.

The deductions made above on the effect of U^* on the heat transfer are more clearly seen in Figs. 5(a) and (b) for Cases A and B, respectively. The effect of the relative velocity is seen to increase with increasing value of U^* for Case A but the opposite is true for Case B. This is consistent with the trend seen in Figs. 4(a) and (b). It is also interesting to note from the figures that the ratio $Nu_{ii}/Nu_{ii(U^*=0)}$ is always greater than unity for $U^* > 0$ and less than unity for $U^* < 0$, whereas the effect of U^* on the ratio $Nu_{oo}/Nu_{oo(U^*=0)}$ is opposite.

The effect of the relative velocity on heat transfer is the opposite of that of the friction factor for Case A but the trend becomes the reverse for Case B. No comparison was made with other works as there are none available in the open literature.

The effects of α and U^* on the influence coefficients, θ_i^* and θ_o^*, are shown in Figs. 6(a) and (b), respectively.

4. CONCLUDING REMARKS

The effects of various parameters such as relative velocity, radius ratio, etc. on the friction factor and

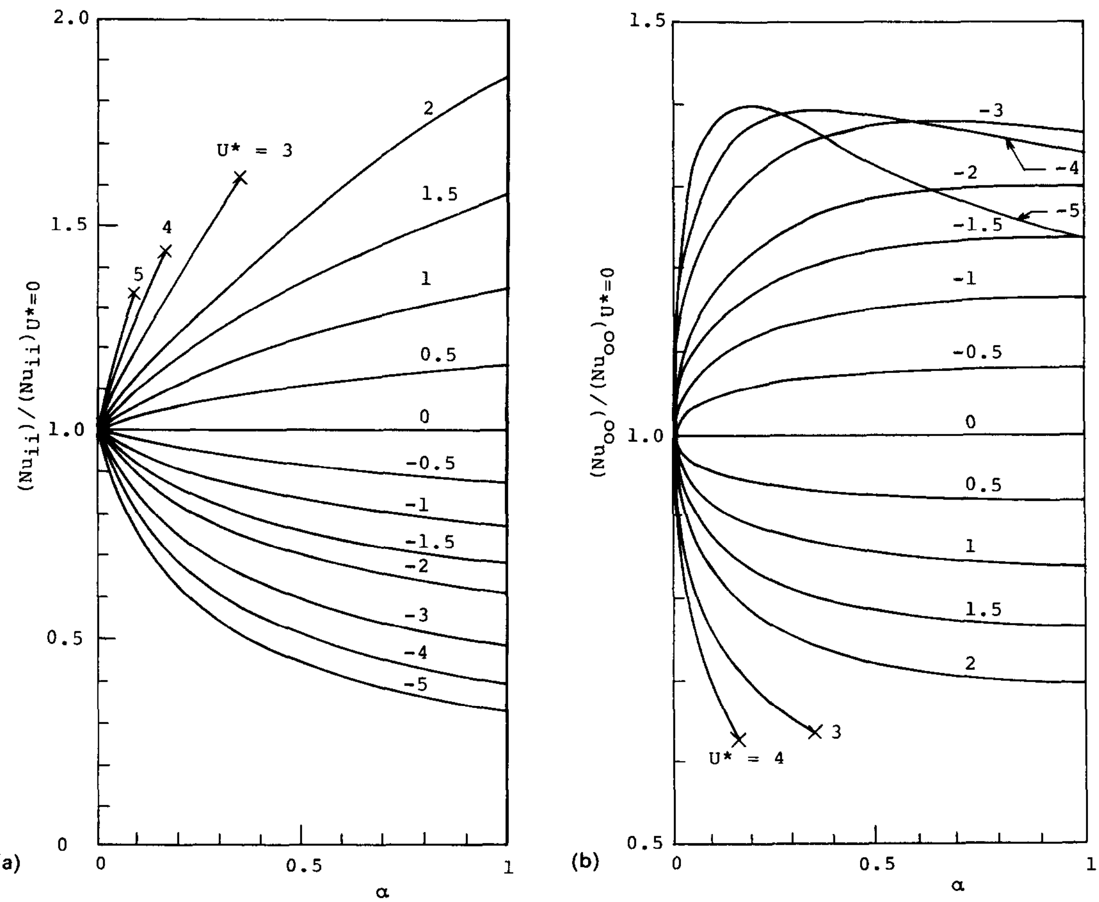

FIG. 5. Nusselt numbers.

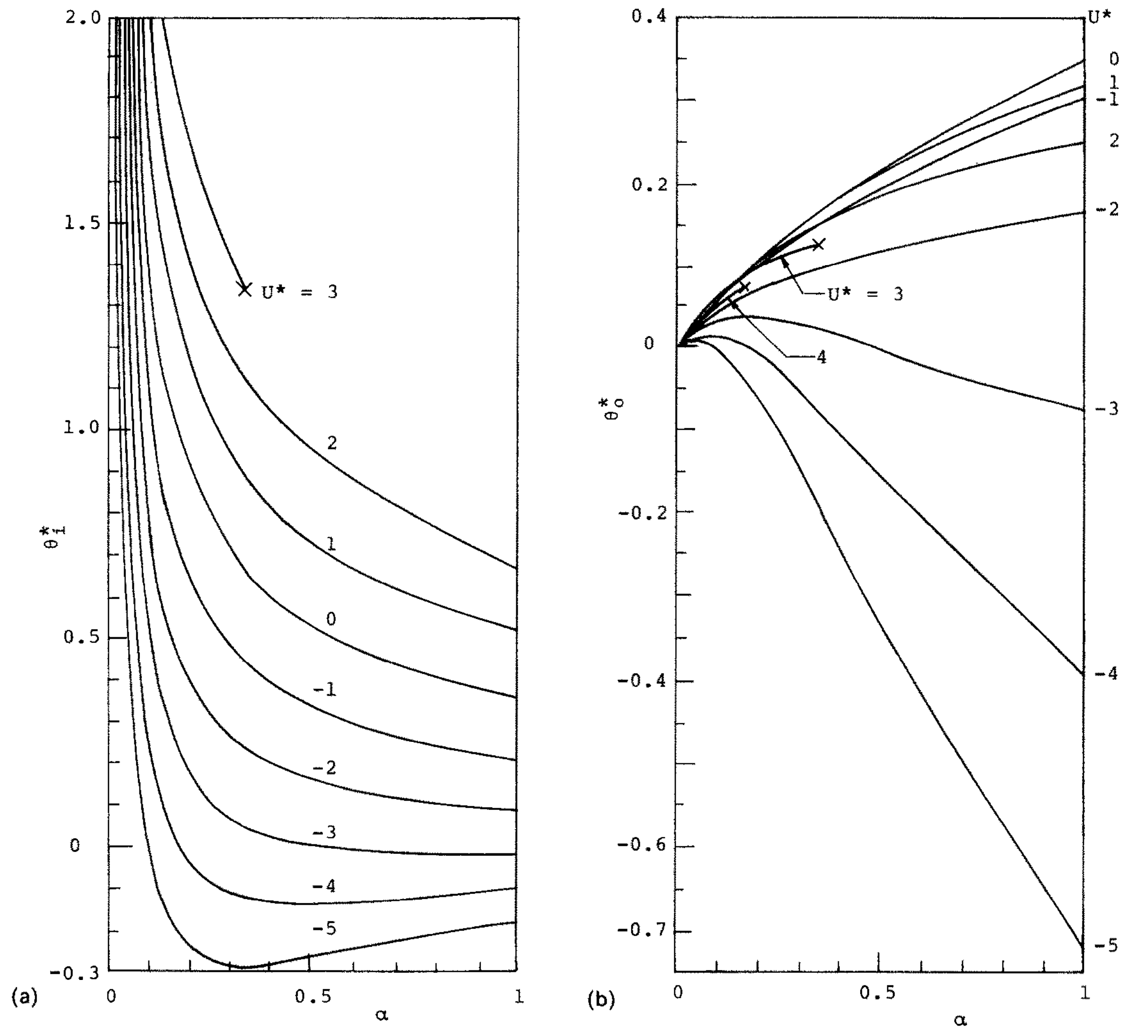

FIG. 6. Influence coefficients.

Nusselt number for laminar flow in concentric annuli with moving cores have been analyzed.

The study showed that for equal conditions, increasing the relative velocity, the following changes were observed:

(a) decrease in friction factor;
(b) increase in Nusselt number for Case A;
(c) decrease in Nusselt number for Case B.

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APPENDIX. THE INFLUENCE COEFFICIENTS, θ_i^* AND θ_o^*

The terms $(T_b - T_{R_i})_{oo}$ and $(T_{R_i} - T_b)_{ii}$ in θ_i^*, equation (19), can be easily obtained from the temperature profile given by equation (8) together with equation (12). The terms q_{R_i} and q_{R_o} can also be readily obtained from the definition of the heat flux, equations (10) and (11). Therefore, the influence coefficients, θ_i^* and θ_o^*, are obtained as

$$\theta_i^* = \frac{Q_{oo} - \alpha^2\left[\frac{2(1-w)}{B} + 1 - (2+B)w + \left\{\frac{(4w-3)}{B} + \left(2w - \frac{1}{4}\right)\right\}\alpha^2 + \frac{(1-2w)}{B}\alpha^4\right]}{\alpha\left[\left(\frac{1}{B} - w\right) - \left(\frac{1}{B} - 1 + wB\right)\alpha^2 + \left(w - \frac{1}{4}\right)\alpha^4 - Q_{ii}\right]} \quad \text{(A1)}$$

$$\theta_o^* = \frac{\alpha\left[Q_{ii} - \left(\frac{3}{4} - wB\right)\right]}{\left(\frac{3}{4} - wB\right) - Q_{oo}} \quad \text{(A2)}$$

where

$$Q_{ii} \equiv \frac{(1-EU^*)}{M}\left[\left\{\frac{25}{24} - \frac{22}{9}wB + \frac{11}{8}w^2B^2\right\} + \left\{-\frac{2(1-w)}{B} + \left(\frac{1}{24} + 4w - 2w^2\right) - \left(\frac{4}{9} + 2w\right)wB + \frac{3}{8}w^2B^2\right\}\alpha^2 + \left\{\frac{(1-2w)}{B} - \left(\frac{17}{24} - w - w^2\right) + \left(\frac{29}{36} - \frac{3}{2}w\right)wB\right\}\alpha^4 + \left(\frac{1}{8} - \frac{5}{6}w + w^2\right)\alpha^6\right] \quad \text{(A3)}$$

$$Q_{oo} = \frac{(1-EU^*)}{M}\left[\left(\frac{7}{24} - \frac{25}{36}wB + \frac{3}{8}w^2B^2\right) + \left\{\left(\frac{43}{24} - \frac{3}{2}w\right) - \left(\frac{31}{9} - 2w\right)wB + \frac{11}{8}w^2B^2\right\}\alpha^2 - \left\{\frac{\{2(1-w) - (1-2w)\alpha^2\}^2}{B} + \left(\frac{47}{24} - 10w + 7w^2\right) - \left(\frac{37}{18} - \frac{11}{2}w\right)wB\right\}\alpha^4 + \left(\frac{3}{8} - \frac{13}{3}w + 7w^2\right)\alpha^6\right]. \quad \text{(A4)}$$

ANALYSE DE L'ECOULEMENT LAMINAIRE ETABLI ET DU TRANSFERT THERMIQUE DANS UN ESPACE ANNULAIRE CONCENTRIQUE AVEC COEUR MOBILE

Résumé—On fait l'analyse du transfert de quantité de mouvement et de chaleur entre un écoulement laminaire établi et un coeur mobile de fluide ou de solide dans une géométrie annulaire. L'espace annulaire est concentrique et les deux parois sont lisses. Le chauffage est à flux uniforme sur une ou les deux surfaces. Les deux solutions fondamentales sont obtenues pour un rapport de flux quelconque, à l'aide de coefficients d'influence. Les effets des différents paramètres tels que la vitesse relative, le rapport des rayons, etc. ... sur le coefficient de frottement et le nombre de Nusselt sont étudiés.

ANALYTISCHE UNTERSUCHUNG EINER VOLL AUSGEBILDETEN LAMINAREN STRÖMUNG MIT WÄRMEÜBERGANG IM KONZENTRISCHEN RINGRAUM MIT BEWEGTEM KERN

Zusammenfassung—Der Impuls- und Wärmeaustausch zwischen einer voll ausgebildeten *laminaren* Strömung in einem Ringspalt und dem bewegten Flüssigkeits- oder Festkörperkern wird betrachtet. Der Ringraum ist konzentrisch, und beide Wandoberflächen sind glatt. An einer oder beiden Oberflächen wird konstante Wärmestromdichte aufgeprägt. Die beiden fundamentalen Lösungen werden berechnet und daraus, mit Hilfe von Einflußkoeffizienten, die Lösung für das innere und äußere Rohr für jedes Verhältnis der Wärmestromdichten. Außerdem wird der Einfluß verschiedener Parameter (Relativgeschwindigkeit, Radienverhältnis usw.) auf den Reibungsbeiwert und die Nusselt-Zahl untersucht.

АНАЛИЗ ПОЛНОСТЬЮ РАЗВИТОГО ЛАМИНАРНОГО ТЕЧЕНИЯ ЖИДКОСТИ И ТЕПЛОПЕРЕНОСА В КОНЦЕНТРИЧЕСКИХ КОЛЬЦЕВЫХ КАНАЛАХ С ДВИЖУЩИМИСЯ ЯДРАМИ

Аннотация—Анализируется перенос количества движения и тепла между полностью развитым ламинарным течением жидкости и движущимся ядром жидкости или твердого тела в кольцевой геометрии. Кольцевой канал является концентрическим и имеет гладкие поверхности обеих стенок. Нагрев осуществляется при однородном тепловом потоке на одной или обеих поверхностях. Получено два фундаментальных решения, позволяющих определить параметры задачи для внутренней и наружной труб при любых отношениях тепловых потоков. Исследуется влияние на коэффициент трения и число Нуссельта таких параметров, как относительная скорость, отношение радиусов и т.д.